CHÖÔNG VIII: SÖ NHIEU XA ANH SANG

I. Nguyen ly Huygens – Fresnel:

Gia sö co mot lo phang (Σ) ñöôc chieu sang bôi nguon ñiem S ñôn sac, böôc song λ. Xet dien tích dσ(P) tren (Σ) tai ñiem P.

Nguyen ly:

- Moi phan tö cua be mat dσ(P) giong nhö mot nguon ñiem ao (nguon thö cap), phat ra song ma bien ño phöc töc thôi tai P tæ le vôi bien ño phöc cua song phat ra tö S tai P, va tæ le vôi dien tích dσ(P).

- Cac nguon ao la ket hôp.

Mot lo trong suot trong mot man ñöôc goi la lo nhieu xa.

$$\underline{s}^*(P,t) = \underline{t}(P)\,\underline{s}_i\,(P,t)$$

$\underline{t}$(P) : ño trong suot phöc = 0 neu khong trong suot tai P

(ham truyen qua) = 1 neu P la mot ñiem cua lo.

$\underline{s}_i$ (P,t) : bien ño song tôi tai P khi khong co lo nhieu xa.

$\underline{s}^*$(P,t) : bien ño song quan sat ñöôc tai P khi khong co nhieu xa, co nghóa la tuan theo cac ñònh luat cua quang hình hoc.

Ghi chu :

- $\underline{t}(P) = -1$ ñoi vôi göông kim loai ly töông.
- $\underline{t}(P) = t_0 e^{-i\frac{2\pi}{\lambda}(n-1)e}$ vôi $t_0 < 1$ ñoi vôi ban thuy tinh be day e.

Bien ño song tai ñiem M phat ra bôi dien tích d σ(P):

$$d\underline{s}_p(M,t) = \underline{f}(P,M).\underline{t}(P).\underline{s}_i(P).e^{i\varphi_{P\to M}}\,d\sigma(P)$$

$\varphi_{P \to M}$: ño lech pha khi truyen tö P tôi M.

$\underline{f}$(P,M) : la mot ham ma ño bien thien cua no rat cham so vôi $e^{i\varphi_{P\to M}}$ o4nhieu xa ñöôc ñat trong moi tröông ñong nhat chiet suat n, neu phöông PM gan vôi phöông cua song tôi, va neu PM lôn hôn nhieu so vôi böôc song, thì song phat ra tö P co dang song cau:

$$\underline{\mathbf{f}}(\mathbf{P},\mathbf{M}) = \frac{\mathbf{C}}{\mathbf{PM}} \qquad \text{C: hang so phöc}$$

Neu M ô kha xaa5, $\underline{f}$(P,M) -> K: hang so phöc, do $\frac{1}{PM}$ thay ñoi khong ñang ke.

II. Nhieu xa FRAUNHOFER.

Ta goi nhieu xa trong cac ñieu kien Fraunhofer la tröông hôp ñac biet khi S va M ô vo cöc. Trong ñieu kien nay, S phat ra song phang vôi vectô song $\vec{k}_i$, gia sö no chieu sang mota5 phang vôi ño trong suot $\underline{t}$(P) = $\underline{t}$(x,y) vuong goc vôi Oz va chöa ñiem O.

Ta ñat: (x,y) la toa ño ñiem P.

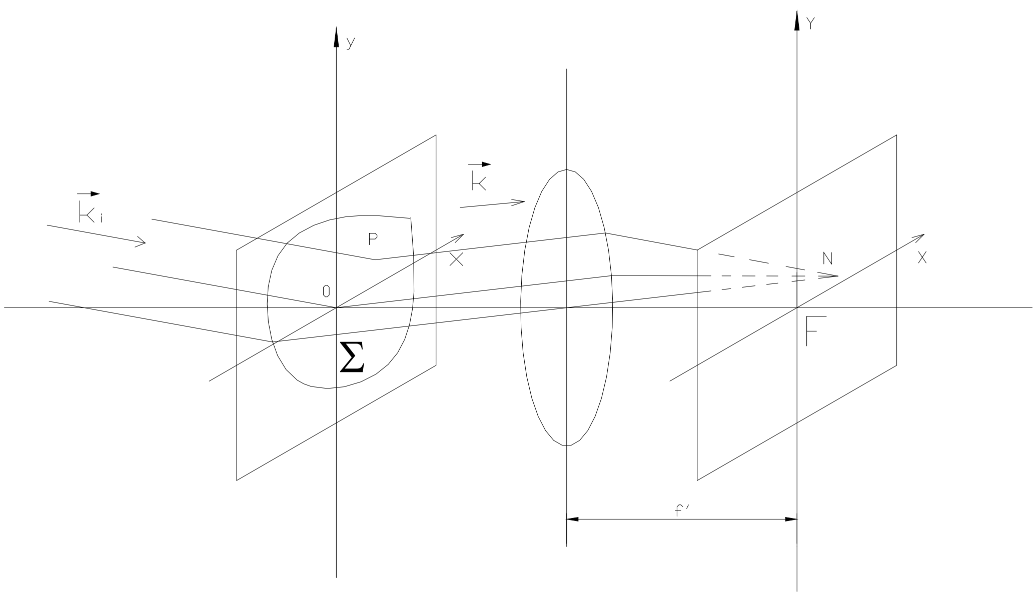
(X,Y) la toa ño ñiem M ñöôc quan sat tren tieu dien cua thau kính.

Moi ñiem M töông öng trong khong gian vat cua thau kính, vôi mot phöông truyen co vectô ñôn vò $\vec{u}(M)$ va vectô song $\vec{k}(M)$.

Gia sö chiet suat moi tröông = 1.

➢ Pha φ_P(M) tai ñiem M cua song thö cap phat ra bôi ñiem P tren (Σ).

(hình 14 trang 156)



Hình 14.Nhieu xa ô vo cöc cua song phang qua lo Σ , man quan sat nam tai tieu dien anh cua mot thau kính

$\varphi_P(M) = \varphi_i(P) + \varphi_{P \to M}$

$\varphi_i(P)$ la pha cua song tôi tai P:

$$\varphi_i(P) - \varphi_i(O) = -\vec{k_i}.\vec{OP}$$

Theo ñònh ly Malus, quang lo (PM) va (HM) bang nhau.

$$(PM) - (OM) = (OH) = -\vec{u}.\vec{OP}$$

va $\varphi_{P \to M} = \varphi_{O \to M} + \vec{k}(M).\vec{OP}$ (sau khi nhan hai ve vôi $-\dfrac{2\pi}{\lambda}$)

$\varphi_O(M) = \varphi_i(O) + \varphi_{O \to M}$: pha tai M cua song thöù cap phat ra tai O.

=> $\varphi_P(M) = \varphi_O(M) + (\vec{k}(M) - \vec{k_i}).\vec{OP}$

➢ <u>Bien ñoi song:</u>

$$\underline{s}(M,t) = \iint_\Sigma d\underline{s}_P(M,t) = Ks_0 e^{i(\omega t + \varphi_0(M))} \iint_\Sigma \underline{t}(x,y) e^{[i(\vec{k}(M) - \vec{k_i})\vec{OP}]} dxdy$$

Bieu dien cac thanh phan cua cac vectô k_i va k(M):

$$k_{ix} = \frac{2\pi}{\lambda}\alpha_i \qquad k_{iy} = \frac{2\pi}{\lambda}\beta_i \qquad k_{iz} = \frac{2\pi}{\lambda}\gamma_i$$

$$k_x = \frac{2\pi}{\lambda}\alpha \qquad k_y = \frac{2\pi}{\lambda}\beta \qquad k_z = \frac{2\pi}{\lambda}\gamma$$

α_i , β_i la cac thanh phan song song vôi (Ox) va (Oy) cua vectô ñôn vò cua phöông song tôi.

α va β la cac thanh phan song song vôi (Ox) va (Oy) cua vectô ñôn vò cua phöông song lo ra tö (Σ) ve phía M.

Neu ta giôi han ô nhöng phöông gan vôi truc:

$$\gamma \approx \gamma_i \approx 1 \qquad \alpha \approx \frac{X}{f'} \qquad \beta \approx \frac{Y}{f'}$$

➢ <u>Be rong cua hình nhieu xa.</u>

Be rong Δx cua lo nhieu xa va be rong goc $\Delta\alpha$ cua hình nhieu xa Fraunhofer tren cung phöông:

$$\Delta x.\Delta\alpha \approx \lambda.$$

➢ <u>Sö dòch chuyen cua lo nhieu xa.</u>

A5dòch chuyen ñöa O ñen O' va P ñen P'.

$$d\underline{s}_{P'}(M) = K's_0 e^{i(\omega t+\varphi_{0'}(M))}\underline{t}(x,y)e^{[i(\vec{k}(M)-\vec{k}_i)\overrightarrow{O'P'}]}d\sigma$$

Söï dòch chuyeån khoâng laøm thay ñoåi vectô $\overrightarrow{k(M)}$ vaø $\overrightarrow{k_i}$.

$\overrightarrow{OP} = \overrightarrow{O'P'}$

Neáu khoâng tính ñeán söï thay ñoåi giöõa K vaø K':

$$d\underline{s}_{P'}(M) = d\underline{s}_P(M)e^{i(\varphi_{0'}(M)+\varphi_0(M))}$$

Sau khi laáy tích phaân treân loã nhieãu xaï:

$$\underline{s}'(M) = \underline{s}(M)e^{i(\varphi_{0'}(M)+\varphi_0(M))}$$

I'(M) = I(M)

=> Bieân ñoä soùng nhieãu xaï taïi moät ñieåm treân tieâu dieän aûnh cuûa thaáu kính chæ chòu ñoä leäch pha gioáng nhau. Cöôøng ñoä cuûa hình nhieãu xaï khoâng thay ñoåi.

- <u>Ñònh lyù Babinet.</u>

Ta goïi hai maøn nhieãu xaï laø phuï nhau neáu toång caùc ñoä trong suoát =1 :

$\underline{t}_1(P) + \underline{t}_2(P) = 1$

<u>Ñònh lyù:</u> Hình nhieãu xaï Fraunhofer cuûa 2 maøn bò phuï thì nhö nhau, tröø aûnh hình hoïc S' cuûa nguoàn S.

$\underline{S}_1(M) + \underline{S}_2(M) = \underline{S}_0(M)$

Neáu M≠S' : $\underline{S}_0(M) = 0 \Rightarrow \underline{S}_1(M,t) = -\underline{S}_2(M,t)$

vaø $I_1(M) = I_2(M)$

III. <u>Nhieãu xaï bôûi loã hình vuoâng.</u>

$\underline{t}(x,y) = 1$ neáu $-\frac{a}{2} < x < \frac{a}{2}$ vaø $-\frac{b}{2} < y < \frac{b}{2}$

$\underline{t}(x,y) = 0$ ôû ngoaøi voøng treân.

$$\underline{s}(M,t) = Ks_0 e^{i(\omega t+\varphi_0(M))}\int_{-\frac{a}{2}}^{\frac{a}{2}}\int_{-\frac{b}{2}}^{\frac{b}{2}} e^{i\frac{2\pi}{\lambda}((\alpha-\alpha_i)x+(\beta-\beta_i)y)}dxdy$$

$$\int_{-\frac{a}{2}}^{\frac{a}{2}} e^{i\frac{2\pi}{\lambda}(\alpha-\alpha_i)x}dx = \frac{e^{i\frac{2\pi}{\lambda}(\alpha-\alpha_i)\frac{a}{2}} - e^{-i\frac{2\pi}{\lambda}(\alpha-\alpha_i)\frac{a}{2}}}{i\frac{2\pi}{\lambda}(\alpha-\alpha_i)\frac{a}{2}} = a\frac{\sin\left[\frac{\pi}{\lambda}\left(\alpha-\alpha_i\right)a\right]}{\frac{\pi}{\lambda}\left(\alpha-\alpha_i\right)a} = a\sin c(u)$$

$$\underline{s}(M,t) = Ks_0 e^{i(\omega t+\varphi_0(M))}\int_{-\frac{a}{2}}^{\frac{a}{2}} e^{i\frac{2\pi}{\lambda}(\alpha-\alpha_i)x}dx\int_{-\frac{b}{2}}^{\frac{b}{2}} e^{i\frac{2\pi}{\lambda}(\beta-\beta_i)y}dy$$

Vôi $u = \frac{\pi}{\lambda}(\alpha-\alpha_i)a$ va $\sin c(u) = \frac{\sin u}{u}$ (ham sinus cardinal)

=> $\underline{s}(M,t) = Ks_0 abe^{i(\omega t+\varphi_0(M))}\sin c(u)\sin c(v)$

vôi $u = \frac{\pi}{\lambda}(\alpha-\alpha_i)a$ va $v = \frac{\pi}{\lambda}(\beta-\beta_i)b$

Cöông ño sang : I(M) = $\underline{s}(M,t).\underline{s}^*(M,t)$

$$I(M) = I_0 \sin c^2(\frac{\pi}{\lambda}(\beta-\beta_i)b).\sin c^2(\frac{\pi}{\lambda}(\alpha-\alpha_i)a)$$

-Hình nhieu xa co tam nam tren phöông cua chum tia tôi

-Cöïc tieu nhieu xa khi:

$\alpha = \alpha_i + p\frac{\lambda}{a}$ hay $\beta = \beta_i + q\frac{\lambda}{b}$ vôi p,q nguyen $\neq 0$

Vet trung tam co ño rong $2\frac{\lambda}{a}$ doc theo (Ox) va $2\frac{\lambda}{b}$ doc theo (Oy)

Cac vet thö cap co ño rong nho hôn hai lan theo cac phöông.

Vet trung tam sang nhat.

- Tröông hôp khe hep.

$\frac{\lambda}{b} \to 0$; β=0.

$$I(M) = I_0 \sin c^2(\frac{\pi}{\lambda}(\theta-\theta_i)a)$$

$$\underline{s}(M,t) = Ks_0 e^{i(\omega t+\varphi_0(M))} \int_{-\frac{a}{2}}^{\frac{a}{2}} e^{i\frac{2\pi}{\lambda}(\theta-\theta_i)x} dx$$

IV. Nhieãu xaï ôû voâ cöïc cuûa soùng phaúng bôûi loã troøn.

Hình nhieãu xaï Fraunhofer cuûa soùng phaúng bôûi loã troøn baùn kính R goàm veát trung taâm hình troøn, taâm laø aûnh hình hoïc cuûa nguoàn, bao quanh laø caùc vaân troøn ñoàng taâm. Caùc vaân troøn coù ñoä saùng giaûm daàn khi caøng xa taâm .

Baùn kính goùc cuûa veát trung taâm coù baäc $\frac{\lambda}{D} : 1{,}22\frac{\lambda}{D}$.

$\Delta\theta = 1{,}22\frac{\lambda}{R}$: ñöôøng kính goùc cuûa veát trung taâm.

V. Nhieãu xaï bôûi taäp hôïp loã nhieãu xaï gioáng nhau.

Khaûo saùt tröôøng hôïp goàm N loã nhieãu xaï gioáng nhau. Loã nhieãu xaï m coù toïa ñoä taâm $O_m(x_m,y_m)$ vaø ñoä trong suoát) vaø ñoä trong suoát $\underline{t}(x,y) = \underline{t}_o(x-x_m,y-y_m)$, haøm $\underline{t}_o$ nhö nhau ñoái vôùi moïi loã nhieãu xaï. Heä ñöôïc chieáu saùng bôûi moät soùng phaúng ñôn saéc coù phöông truyeàn ñöôïc cho bôûi α_i vaø β_i .

$$\underline{s}_m(M,t) = Ks_0 e^{i(\omega t+\varphi_{0m}(M))} \iint_{\Sigma m} \underline{t}(x,y) e^{[i\frac{2\pi}{\lambda}((\alpha-\alpha_i)(x-x_m)+(\beta-\beta_i)(y-y_m))]} dxdy$$

Ñoåi bieán: $\xi = x - x_m$ vaø $\eta = y - y_m$

$$\underline{s}_m(M,t) = Ks_0 e^{i(\omega t+\varphi_{0m}(M))} \iint_{\Sigma m} \underline{t}_0(\xi,\eta) e^{[i\frac{2\pi}{\lambda}((\alpha-\alpha_i)\xi+(\beta-\beta_i)\eta)]} d\xi d\eta$$

tích phaân $$F_D(M) = \iint_{\Sigma m} \underline{t}_0(\xi,\eta) e^{[i\frac{2\pi}{\lambda}((\alpha-\alpha_i)\xi+(\beta-\beta_i)\eta)]} d\xi d\eta$$

nhö nhau ñoái vôùi moïi loã nhieãu xaï, ñöôïc goïi laø soá haïng nhieãu xaï.

Theo nguyeân lyù Huygens – Fresnel, N loã nhieãu xaï ñöôïc chieáu saùng moät caùch keát hôïp, bieân ñoä taïi M ôû voâ cöïc baèng toång caùc bieân ñoä nhieãu xaï bôûi töøng loã.

$$\underline{s}(M,t) = Ks_0.F_D(M)e^{i\omega t}\sum_{m=1}^{N} e^{i\varphi_{0m}(M)}$$

φ_{0m} : pha tai M cua song thöi cap phat ra töi O_m

$\varphi_{Om}(M) = \varphi_O(M) + (\overrightarrow{k}(M) - \overrightarrow{k_i}).\overrightarrow{OO_m}$

$$= \varphi_0(M) + \frac{2\pi}{\lambda}\left[(\alpha - \alpha_i)x_m + (\beta - \beta_i)y_m\right]$$

Tong Σ la <u>so hang giao thoa</u> cua N song nhieu xa bôi N lo nhieu xa.

$$F_I(M) = \sum_{m=1}^{N} e^{i\frac{2\pi}{\lambda}\left[(\alpha-\alpha_i)x_m + (\beta-\beta_i)y_m\right]}$$

- Tröông hôp phan bo ngau nhien.

  Khao sat tröông hôp N rat lôn.

$$I = I_0\left|F_D(M)\right|^2\left|F_I(M)\right|^2$$

$$\left|F_I(M)\right|^2 = \left(\sum_{m=1}^{N} e^{i\varphi_m}\right)\left(\sum_{m=1}^{N} e^{-i\varphi_m}\right)$$

vôi $\varphi(M) = \dfrac{2\pi}{\lambda}\left[(\alpha - \alpha_i)x_m + (\beta - \beta_i)y_m\right]$

$$\left|F_I(M)\right|^2 = N + \sum_m \sum_{n\neq m} e^{i(\varphi_n - \varphi_m)}$$

$$\sum_m \sum_{n\neq m} e^{i(\varphi_n - \varphi_m)} = \sum_{n>m}\left(e^{i(\varphi_n-\varphi_m)} - e^{-i(\varphi_n-\varphi_m)}\right)$$

$$\Rightarrow \left|F_I(M)\right|^2 = N + 2\sum_{n>m}\cos\left[\varphi_n(M) - \varphi_m(M)\right]$$

Neu cac lo nhieu xa ñöôc phan bo ngau nhien thì cac goc $\varphi_{nm}(M) = \varphi_n(M) - \varphi_m(M)$ cung ñöôc phan bo ngau nhien va tong $\sum_{n>m}\cos\left[\varphi_n(M) - \varphi_m(M)\right]$ chæ khac khong ñoi vôi α va β rat gan α_i va β_i.

Tong nay chöa $\dfrac{N(N-1)}{2}$ so hang:

$\left|F_I(M)\right|^2 = N^2$ ô phöông cua song tôi

$\left|F_I(M)\right| = N$ cac phöông khac.